Sensitizing Complex Hamiltonian for Study of Real Spectra .

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Abstract

We notice many complex Hamiltonians yield real spectra .Interestingly some of them belong to $\mathcal{PT}$ symmetry and others non-$\mathcal{PT}$ symmetry in nature.In the case of simple quantum systems, one can calculate the energy spectra analytically, however in other cases one has seek numerical results .We give specific attention on stable real spectra .

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Key words : Complex quantum systems , $\mathcal{PT}$ symmetry,non-$\mathcal{PT}$symmetry,real stable spectra.

In the study of spectra ,mainly stability plays the major criteria.Spectral stability in quantum systems is mainly associated with Hermiticity , $\mathcal{PT}$ symmetr and pseudo-Hermiticity [1-3].In fact the term Hermiticity should more appropriately be stated as $\mathbf{self-adjoint}$ oprator.However in this paper we would like focus our attention on citing few examples on complex quantum systems that yield stable real spectra-We categorize the the quantum systems in two groups :(i) analytical results (ii) numerical results. **Analytical Calculation.**

$$H = p^2 + x^2 + ix \tag{1}$$

This Hamiltonian is a simple Harmonic Oscillator under the influence of a $\mathcal{PT}$ perturbation .The energy eigenvalue is

$$E_n = 2n + \frac{5}{4} \tag{2}$$

Now consider a slight different complex system,whixh is complex but not $\mathcal{PT}$ symmetry in its nature.

$$H = p^2 + x^2 + x + ip \tag{3}$$

In this case the energy eigenvalue is

$$E_n = 2n + 1 \tag{4}$$

The interesting feature in above consideration ,excites one to propose a different complex oscillator as

$$H = (1 + i\lambda)p^2 + (1 - i\lambda)x^2 \tag{5}$$

In this case the energy eigenvalue [4-6] is

$$E_n = \sqrt{(1 + \lambda^2)}(2n + 1) \tag{6}$$

Interestingly if one slightly introduces a different complex oscillatior as

$$H = (1 + i\lambda + e^{i\lambda}p^2 + (1 - i\lambda + e^{-i\lambda})x^2 \tag{7}$$

In this case the energy eigenvalue[4-6] is

$$E_n = \sqrt{(2 + \lambda^2 + 2cos(\lambda) + 2\lambda sin(lambda))}(2n + 1) \tag{8}$$

Numerical Calculation. Here we focus our attention on numerical results .For numerical results we use matrix diagonalisation method(MDM)[7] as follows.The eigenvalue relation is

$$H\Psi = E\Psi \tag{9}$$

where

$$\Psi = sum_m A_m|\phi m \tag{10}$$

where ϕ_m stands for Harmonic Oscillator wave function. Let us consider a simple $\mathcal{PT}$ symmetry Hamiltonian which is a slight deviation of Eq(1) as

$$H = p^2 + x^2 + e^{-ix} \tag{11}$$

The eigen values are stable and real and reflected in table-I.In this context we would like to state that the $\mathcal{PT}$ symmetry oscillator

$$H = p^2 + x^2 + 10e^{-ix} \tag{12}$$

bears no stable real eigenvalues but complex. Now consider another another $\mathcal{PT}$ symmetry Hamiltonian as

$$H = p^2 + x^2 + e^{ixp} + e^{-ipx} \tag{13}$$

Interestingly the MDM reflects unstable real values ,which varies with the size of the matrix .However ,The Hamiltonian

$$H = p^2 + x^2 + e^{ixp} + e^{-ipx} + x^4 \tag{14}$$

reflects real stable eigenvalues and are cited in table-I. Lastly we consider a complex Oscillator with x^4 term as

$$H = p^2 + x^2 + x^2p^2 + x^4 + x + ip \tag{15}$$

In this case eigen values are not only real but also stable and are cited in table-I.In conclusion we notice that complex hamiltonians also yield real stable eigenvalues like real Hamiltonians .Following above procedures one can propose many non-Hermitian quantum systems.

References

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Table-I

Energy levels of complex Quantum systems .

Hamiltonian	levels	Eigenvalues
$H = p^2 + x^2 + e^{-ix}$	**0**	**1.996 272 0**
	1	**3.306 160 5**
	2	**5.014 526 6**
	3	**6.827 908 0**
$H = p^2 + e^{ixp} + e^{-ipx} + x^4$	**0**	**1.998 495**
	1	**4.360 294**
	2	**7.524 852**
	3	**11.347 845**
$H = p^2 + x^2 + x^2p^2 + x^4 + x + ip$	**0**	**1.341 961 0**
	1	**5.830 157 6**
	2	**11.644 376 4**
	3	**18.729 604 3**